FINAL EXAMINATION

Dec. 8, 2009, ~~10~~ 2 – ~~1~~1:50 ~~am~~ p

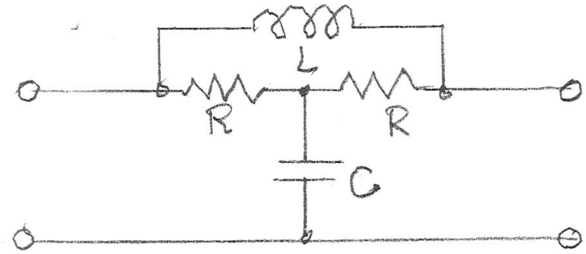
ECE 580

Prof. G. Temes **Open Book**

1. The two-port shown operates between 50 ohm terminations, and $S_{11} = 0$ at all frequencies. $C = 1$ pF.

   (a) Find R and L.

   (b) Find S_{12} as a function of frequency.



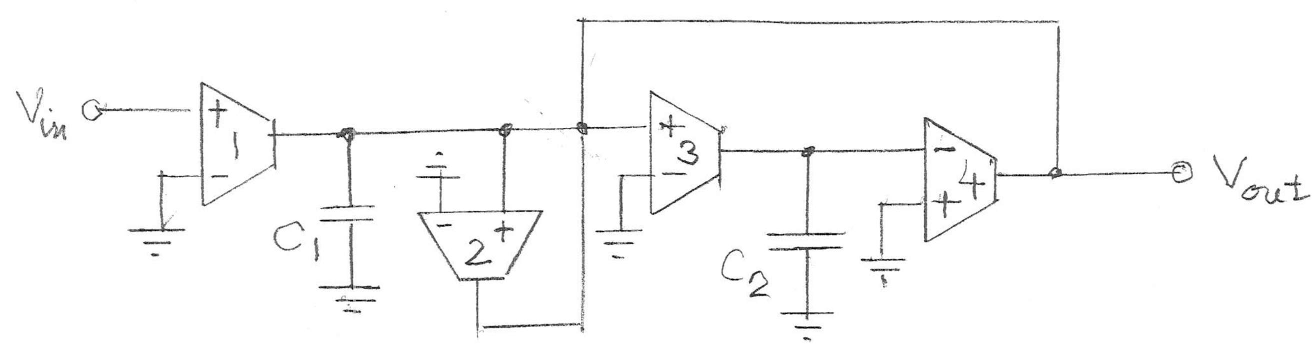
2. The -3-dB frequency of a 12th-order Butterworth filter is $f_3 = 10$ MHz.

   (a) Where are the dominant poles?

   (b) What is the dominant pole Q_p?

3. (a) Find the transfer function of the G_m-C filter shown.

   (b) Let the swing at the output of G_{m3} be 50% of the allowed maximum. How can it be scaled back to optimum?

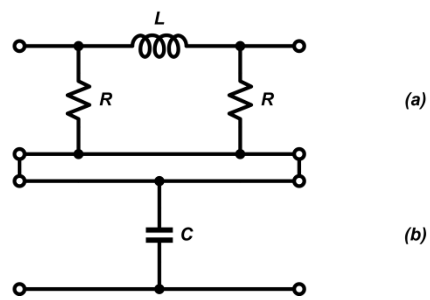


ECE 580 Fall 2009

Final Examination Solution

1.

(a)



$$For \quad S_{11} = 0 \quad , Z_1 = R_{in}$$

$$For \quad I_1 = 1A$$

$$V_1 = Z_1 = Z_{11}I_1 + Z_{12}I_2 = Z_{11} - Z_{12}\frac{V_2}{R_{in}}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 = Z_{12} - Z_{22}\frac{V_2}{R_{in}}$$

$$\Rightarrow V_2 = \frac{Z_{12}}{1 + \frac{Z_{22}}{R_{in}}}$$

$$\Rightarrow Z_1 = Z_{11} - \frac{\frac{Z_{12}{}^2}{R_{in}}}{1 + \frac{Z_{22}}{R_{in}}} = \frac{Z_{11}R_{in} + \det Z}{R_{in} + Z_{22}} = R_{in}$$

$$For \quad Z_{11} = Z_{22}$$

$$R_{in}{}^2 + Z_{11}R_{in} = Z_{11}R_{in} + Z_{11}{}^2 - Z_{12}{}^2 \quad - - - - - (1)$$

$$Z_a = \begin{bmatrix} \frac{sLR + R^2}{sL + 2R} & \frac{R^2}{sL + 2R} \\ \frac{R^2}{sL + 2R} & \frac{sLR + R^2}{sL + 2R} \end{bmatrix} \qquad Z_b = \begin{bmatrix} \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & \frac{1}{sC} \end{bmatrix} \qquad Z = Z_a + Z_b = \begin{bmatrix} \frac{1}{sC} + \frac{sLR + R^2}{sL + 2R} & \frac{1}{sC} + \frac{R^2}{sL + 2R} \\ \frac{1}{sC} + \frac{R^2}{sL + 2R} & \frac{1}{sC} + \frac{sLR + R^2}{sL + 2R} \end{bmatrix}$$

$$Use \quad (1) \Rightarrow R_{in}{}^2 = Z_{11}{}^2 - Z_{12}{}^2$$

$$\Rightarrow R_{in}{}^2 = \left(\frac{sLR}{sL+2R}\right)\left(\frac{2}{sC} + \frac{sLR+2R^2}{sL+2R}\right) \qquad \Rightarrow \qquad R_{in}{}^2 = \frac{sCLR^2+2LR}{2RC+sLC}$$

$$sCLR^2 + 2LR = sCLR_{in}{}^2 + 2R_{in}{}^2 RC$$

$$\Rightarrow R = R_{in}$$

$$\Rightarrow L = R_{in}{}^2 C$$

$R = 50\Omega$

$L = 2.5nH$

(b)

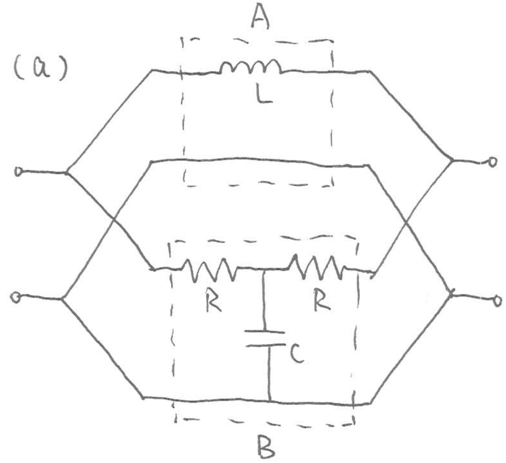
$$V_1 = \frac{E_1}{2} = Z_{11}\frac{E_1}{2R_{in}} - Z_{12}\frac{V_2}{R_{in}}$$

$$V_2 = \frac{E_1}{2Z_{12}}(Z_{11} - R_{in})$$

$$S_{12} = S_{21} = \frac{V_2}{\frac{E_1}{2}} = \frac{Z_{11} - R_{in}}{Z_{12}}$$

$$\Rightarrow S_{12} = \frac{s(L - R^2C) + 2R}{s(L + R^2C) + 2R} = \frac{1}{sRC+1} = \frac{1}{1 + j\omega 5E^{-11}}$$

1. (a)



$$Y_A = \begin{bmatrix} \frac{1}{SL} & -\frac{1}{SL} \\ -\frac{1}{SL} & \frac{1}{SL} \end{bmatrix} \qquad Y_B = \begin{bmatrix} \frac{SC+\frac{1}{R}}{SRC+2} & -\frac{1}{R}\frac{1}{SRC+2} \\ -\frac{1}{R}\frac{1}{SRC+2} & \frac{SC+\frac{1}{R}}{SRC+2} \end{bmatrix}$$

$$Y = Y_A + Y_B$$

$$S_{11} = 0 \Rightarrow Z_{in} = 50\,\Omega$$

$$I_2 = y_{21}V_1 + y_{22}V_2 = -\frac{V_2}{R_2} \Rightarrow V_2 = -\frac{y_{21}}{\frac{1}{R_2}+y_{22}}V_1$$

$$\therefore I_1 = y_{11}V_1 + y_{12}V_2 = y_{11}V_1 - \frac{y_{12}y_{21}}{\frac{1}{R_2}+y_{22}}V_1$$

$$\Rightarrow Z_{in} = \frac{V_1}{I_1} = \frac{1/R_2 + y_{22}}{y_{11}^2 - y_{12}^2 + y_{11}/R_2} = 50\,\Omega$$

$$\Rightarrow y_{11}^2 - y_{12}^2 = \frac{1}{R_2^2} \Rightarrow (y_{11}-y_{12})(y_{11}+y_{12}) = \left(\frac{2}{SL} + \frac{SRC+2}{R(SRC+2)}\right)\left(\frac{SRC}{R(SRC+2)}\right) = \frac{1}{50^2}$$

$$\Rightarrow SLCR_2^2 + 2RCR_2^2 = SR^2LC + 2RL$$

$$\therefore \left.\begin{matrix} R^2 = R_2^2 \\ L = CR_2^2 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} R = 50\,\Omega \\ L = 2.5\,nH \end{matrix}\right.$$

(b) Symmetric network $\Rightarrow S_{22} = 0$,

$$S_{12} = \frac{b_1}{a_2}, \quad E_1 = 0.$$

Matching $\Rightarrow b_1 = \frac{V_1}{\sqrt{R_1}}, \quad a_2 = \frac{V_2}{\sqrt{R_2}}$

$$I_1 = y_{11}V_1 + y_{12}V_2 = -\frac{V_1}{R_1} \Rightarrow V_1\left(y_{11}+\frac{1}{R_1}\right) = -V_2 y_{12}$$

$$\therefore S_{12} = \frac{b_1}{a_2} = \frac{V_1}{V_2} = -\frac{y_{12}}{y_{11}+\frac{1}{R_1}} = \frac{1}{SRC+1} = \frac{1}{1+j\omega 5\times 10^{-11}}$$

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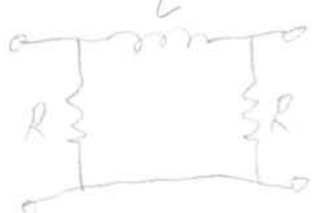
1. (a)



$$\underset{\sim}{Z_A} = \begin{bmatrix} \frac{R\cdot(R+sL)}{2R+sL} & \frac{R^2}{2R+sL} \\ \frac{R^2}{2R+sL} & \frac{R\cdot(R+sL)}{2R+sL} \end{bmatrix}$$

$$\underset{\sim}{Z_B} = \begin{bmatrix} \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & \frac{1}{sC} \end{bmatrix}$$

$$\underset{\sim}{Z} = \underset{\sim}{Z_A} + \underset{\sim}{Z_B}$$

$$Z_{11} = Z_{22} = \frac{1}{sC} + R - \frac{R^2}{2R+sL} \quad \cdots\cdots \text{①}$$

$$Z_{12} = Z_{21} = \frac{1}{sC} + \frac{R^2}{2R+sL} \quad \cdots\cdots \text{②}$$

for $I_1 = 1A$

$V_1 = Z_1 = Z_{11}I_1 + Z_{12}I_2 = Z_{11} - Z_{12}V_2/r$

$V_2 = Z_{21}I_1 + Z_{22}I_2 = Z_{12} - Z_{22}V_2/r$

$$Z_1 = \frac{Z_{11}r + \det \underset{\sim}{Z}}{r + Z_{22}}$$

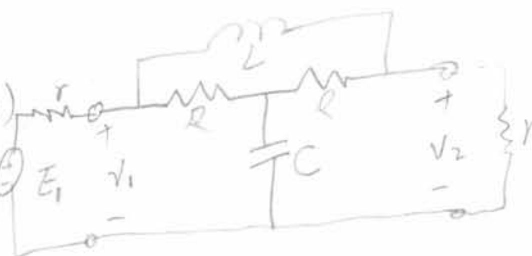
when [illegible] $= 0 \Rightarrow Z_1 = r \Rightarrow r^2 = Z_{11}^2 - Z_{12}^2 \quad \cdots\cdots$ ③

put ①. ② into ③

$$\left(\frac{2}{sC}+R\right)\left(R - \frac{2R^2}{2R+sL}\right) = r^2 \Rightarrow 2RLs + R^2LCs^2 = r^2 2RCs + r^2LCs^2$$

$$\Rightarrow \begin{cases} 2RL = r^2 \cdot 2RC \\ R^2 LC = r^2 LC \end{cases} \Rightarrow \begin{aligned} R &= r = 50\,\Omega \\ L &= r^2 \cdot C = 2.5\,nH \end{aligned}$$

(b)



$$V_1 = \frac{E_1}{2} = Z_{11} \cdot \frac{E_1}{2r} - Z_{12} \frac{V_2}{r}$$

$$V_2 = \frac{E_1}{2 Z_{12}} \left[Z_{11} - r \right]$$

So $$S_{12} = S_{21} = \frac{V_2}{E_1/2} = \frac{Z_{11} - r}{Z_{12}} = \frac{2R + S(L - R^2 C)}{2R + S(L + R^2 C)} = \frac{100}{100 + 5 \times 10^{-9} \cdot S}$$

$$S_{12}(j\omega) = \frac{100}{100 + j \cdot 5 \times 10^{-9} \omega}$$

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2. (a)

$$\omega_{3dB}{}^2 = \sigma_p{}^2 + \omega_p{}^2 \Rightarrow \omega_{3dB} = \sqrt{\sigma_p{}^2 + \omega_p{}^2} = 2\pi 10E^6 = 6.28E^7 \quad [\frac{rad}{s}]$$

$$\sigma_p = \sin(\beta)\omega_{3dB} = 8.20E^6$$

$$\omega_p = \pm 6.23E^7$$

$$dominate \quad pole(p1, p2) = -8.20E^6 \pm j6.23E^7 \quad [\frac{rad}{s}]$$

(b)

$$S_k = C_n^{-\frac{1}{2n}} e^{\frac{j\pi(n-1+2k)}{2n}}$$

$$S_p = \sigma_p + j\omega_p \qquad \Rightarrow Q = \frac{|S_p|}{2|\sigma_p|}$$

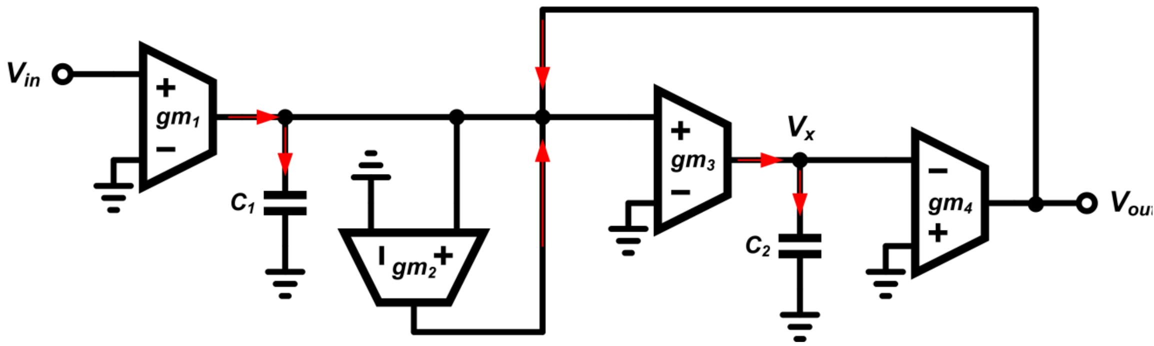
$$Q = \frac{C_n^{-\frac{1}{2n}}}{2\left|C_n^{-\frac{1}{2n}} \cos(\frac{\pi(n-1+2k)}{2n})\right|} = \frac{1}{2\left|\cos(\frac{\pi(n-1+2k)}{2n})\right|}$$

$$Q_{\max} \quad happens \quad at \quad k = 1$$

$$\Rightarrow Q = \frac{1}{2\left|\cos(\frac{\pi(12-1+2)}{24})\right|} = 3.83$$

3.

(a)



$KCL\ @\ Node\ V_{out}$

$$gm_1V_{in} - V_{out}sC_1 + gm_2V_{out} - V_xgm_4 = 0 \quad ---(1)$$

$KCL\ @\ Node\ V_X$

$$gm_3V_{out} = V_XsC_2$$

$$V_X = \frac{gm_3V_{out}}{sC_2} \quad -----------------(2)$$

$Put\ (2)\ in\ (1)$

$$gm_1V_{in} - V_{out}sC_1 + gm_2V_{out} - \frac{gm_3gm_4V_{out}}{sC2} = 0$$

$$\Rightarrow H(s) = \frac{sgm_1C_2}{s^2C_1C_2 - sgm_2C_2 + gm_3gm_4}$$

Note: The circuit is unstable

(b)

If V_x is 50% of the allowed maximum, multiply gm_3 by 2 and divide gm_4 by 2 in order to make sure the feedback current the same.